Subject: More mathematical functions
From: Van Snyder
Reference: 03-258r1, section 2.4.4.3, 04-184r1

1 Number

TBD

2 Title

More mathematical functions.

3 Submitted By

J3

4 Status

For consideration.

5 Basic Functionality

More mathematical functions.

6 Rationale

Mathematical functions for complex type are occasionally needed. The only ones that are available for complex type are ABS, COS, EXP, LOG and SIN. The other mathematical functions that are provided for real type are useful in practice for complex type as well. Inverse hyperbolic functions and other functions are useful. Simple identities for complex argument exist, but it is a burden to expect users to look them up, and processors might be able to produce more efficient implementations. For inverse hyperbolic functions, there are simple identities involving square root and logarithm, but these can have substantial cancellation error for some ranges of values, so it is important to be careful in their implementation. Processors would presumably include careful intrinsic implementations of these functions.

7 Estimated Impact

Minor but tedious. Estimated at meeting 169 to be 4 on the JKR scale.

8 Detailed Specification

Provide ACOS, ASIN, ATAN, COSH, SINH, TAN and TANH for complex type. Provide inverse hyperbolic functions, including for complex type. In the case of TAN, specify that the real part of the argument is regarded as a value in radians.

It is proposed at this time that only the above named existing intrinsic functions be extended to complex type, that inverse hyperbolic functions be provided for real and complex arguments, and that no additional new functions be introduced, at last not in the context of this proposal. The following two paragraphs are included from 04-184r1 for reference purposes only.

The following also appear in applications, and have better round-off characteristics for x near zero when implemented directly rather than as written here: $e^x - 1$, $\log(x+1)$, $x - \log(x+1)$, $(x - \sin(x))/x^3$, $(1 - \cos(x))/x^2$, $(\sinh(x) - x)/x^3$, $(\cosh(x) - 1)/x^2$ and $1/\Gamma(x+1) - 1$. The function $x - 1 - \log(x)$ has better round-off characteristics for x near one when implemented directly rather than as written

here. These should be provided for both real and complex arguments. The first two are the ones most commonly found in applications.

A few other functions are useful, especially $\Gamma(x)$, erf(x), erfc(x) and $\exp(x^2)$ erfc(x). These are sufficiently difficult to do well for complex arguments that the standard should not require it.

8.1 Suggested edits

The following edits are proposed for the purpose of indicating the scope of the project.

ACOSH(X)	Inverse hyperbolic cosine	294:25+
ASINH(X)	Inverse hyperbolic sine	294:26+
ATANH(X)	Inverse hyperbolic tangent	294:27+

[Editor: Add the following three items to the list in alphabetical order:] 298:16

ACOSH	ACOSH	default real
ASINH	ASINH	default real
ATANH	ATANH	default real

[Editor: after "1" insert ", or of type complex".] 301:10

13.7.3$\frac{1}{2}$ ACOSH (X) 301:14+

Description. Inverse hyperbolic cosine function.

Class. Elemental function.

Argument. X shall be of type real or complex.

Result Characteristics. Same as X.

Result Value. The result has a value equal to a processor-dependent approximation to the inverse hyperbolic cosine function of X.

Example. ACOSH (1.5430806) has the value 1.0 (approximately).

[Editor: after "1" insert ", or of type complex".] 304:14

13.7.12$\frac{1}{2}$ ASINH (X) 304:18+

Description. Inverse hyperbolic sine function.

Class. Elemental function.

Argument. X shall be of type real or complex.

Result Characteristics. Same as X.

Result Value. The result has a value equal to a processor-dependent approximation to the inverse hyperbolic sine function of X.

Example. ASINH (1.1752012) has the value 1.0 (approximately).

[Editor: after "real" insert "or complex".] 305:31

13.7.15$\frac{1}{2}$ ATANH (X) 306:13+

Description. Inverse hyperbolic tangent function.

Class. Elemental function.

Argument. X shall be of type real or complex.

Result Characteristics. Same as X.

Result Value. The result has a value equal to a processor-dependent approximation to the inverse hyperbolic tangent function of X.

Example. ATANH (0.76159416) has the value 1.0 (approximately).

1	[Editor: after “real” insert “or complex”.]	309:7
2	[Editor: after “real” insert “or complex”.]	352:15
3	[Editor: after “real” insert “or complex”.]	355:16
4 5	[Editor: “, with X . . . radians” ⇒ “. If X is of type real, it is regarded as a value in radians. If X is of type complex, its real part is regarded as a value in radians”.]	355:18-19
6	[Editor: after “real” insert “or complex”.]	355:24

7 9 History